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EXPERIMENTAL EDDY CURRENT MEASUREMENTS OF FLAWED EDGES COMPARED WITH RESULTS FROM PROBABILISTIC NUMERICAL MODELS (PREPRINT)

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14. ABSTRACT
Eddy current detection of flaws in edges presents challenges in experimental procedures during benchmark studies in the laboratory for model validation as well as practical implementation of a real world detection system. These difficulties result in distortions to the signal that mask the effects from the flawed region itself. Rather than attempting to perfect the experimental setup, we propose to make the numerical models more robust by incorporating randomness in the experimental procedure with uncertainty quantification methods. We present the motivation for the specific method chosen, the probabilistic collocation method (PCM), and the mathematical development behind the method, and then present the results from numerical simulations with a validation measure.

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Experimental Eddy Current Measurements of Flawed Edges Compared with Results from Probabilistic Numerical Models

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Abstract. Eddy current detection of flaws in edges presents challenges in experimental procedures during benchmark studies in the laboratory for model validation as well as practical implementation of a real world detection system. These difficulties result in distortions to the signal that mask the effects from the flawed region itself. Rather than attempting to perfect the experimental setup, we propose to make the numerical models more robust by incorporating randomness in the experimental procedure with uncertainty quantification methods. We present the motivation for the specific method chosen, the probabilistic collocation method (PCM), and the mathematical development behind the method, and then present the results from numerical simulations with a validation measure.

Keywords: probabilistic, collocation, eddy, current

1. Introduction

In numerical modeling of eddy-current testing (ECT), geometry of the component and material parameters are typically set with some nominal value that has been determined from an average of several different readings. In certain cases, information obtained from these types of deterministic models is sufficient, but in general the parameters of the problem cannot be simply represented as a single value and must be treated as a random variable due to inherent variability. This implies that the response from the model is also a random quantity with its own uncertainty characteristics. Uncertainty quantification (UQ) methods for forward models calculate the uncertainty distributions of outputs from the models given random inputs. There are many such stochastic methods [4], and each has advantages and disadvantages, depending on the forward problem to which they are applied and the information needed. In nondestructive evaluation (NDE) applications and specifically for ECT problems, the output information from stochastic forward models is used largely for stochastic inversion methods and model assisted probability of detection (MAPOD) studies, both of which can rely heavily on a Bayesian framework. Because of this, the UQ method should determine the full probability distribution function (PDF) of the response. Since there are already several different commercial codes available for ECT simulation [1, 2], the UQ method should be non-intrusive. A method that is non-intrusive does not require reworking

of the internal code of the numerical simulation and essentially treats the forward model as a block box with random inputs. The classical non-intrusive methods for determining the full uncertainty characteristics of the response from forward models are Monte Carlo (MC) methods. The random input space is sampled randomly and the model is simulated at each sample, eventually resulting in the full PDF of the response. Even though there exist techniques to reduce the amount of samples needed by selectively choosing areas of the input space to sample [4], these methods can still require quite a few model solutions and are not feasible for any forward model of reasonable size. To reduce the time of such a simulation, the forward model can be represented by some surrogate model that takes much less time to solve. In this work, we use the probabilistic collocation method (PCM) to construct a surrogate model and produce the full PDF of the response by performing a classical MC simulation on this surrogate. This surrogate model was shown to converge to the forward model at third order in [11]. The flawed edge problem is presented, and a forward model for this problem is formulated and validated. Results from the development of the surrogate model are shown along with some first moment validation measures. The full PDF of the change of resistance and reactance of an eddy current coil are then produced, and problems with the method going forward are then discussed.

2. Definitions

In the paper, Z is a random variable, where lower case z is a specific outcome of the random variable. Similarly R is a random vector, where r is the specific outcome of the vector.

3. PCM Development

The basics of the PCM are shown here. For further details of the development, see the text by Xiu. [7]

3.1. Orthogonal Polynomials

The gPC basis for the distribution of a random variable, Z, is defined as a sequence of polynomials,

$$\varphi_i(z), i = 1, \ldots, n$$

that satisfy the orthogonality condition:

$$\left(\varphi_i(z), \varphi_j(z)\right)_p = \begin{cases} \|\varphi_i(z)\|^2, & i = j \\ 0, & i \neq j \end{cases} \tag{1}$$

Here, the inner product:

$$\left(\varphi_i(z), \varphi_j(z)\right)_p = \int_{\Omega} \varphi_i(z)\varphi_j(z)\, p(z)dz \tag{2}$$

where $p(z)$ is the density function of the random variable.

3.2. *Arbitrary Distributions*

Orthogonal polynomials defined in the previous sections have known type for standard distributions such as normal or uniform. As stated earlier, there is a wealth of distributions encountered in NDE, and to compensate for all of these, a more general method of developing gPC expansions is needed. This can be accomplished with the well known three term recurrence relation [5, 6]:

$$
\begin{aligned}
&\varphi_0(x) = 1 \\
&\varphi_1(x) = x - \frac{(x\varphi_0, \varphi_0)_p}{\|\varphi_0\|_p^2} \\
&\vdots \\
&\varphi_n = (x - A_n)\varphi_{n-1}(x) - B_n\varphi_{n-2}(x), \;\; n = 2,3, \dots \\
&A_n = \frac{(x\varphi_{n-1}, \varphi_{n-1})_p}{\|\varphi_{n-1}\|_p^2} \\
&B_n = \frac{(x\varphi_{n-1}, \varphi_{n-2})_p}{\|\varphi_{n-2}\|_p^2}
\end{aligned}
\tag{3}
$$

This three term recurrence relation has been shown to produce orthogonal polynomials with respect to the density function, p. This enables us to define a distribution numerically and perform one dimensional numerical integration [8] to determine the orthogonal polynomials for any arbitrary distribution.

3.3. *Function of a Random Variable*

Once the orthogonal basis for the random variable has been determined, we define a function, $f(z)$, of that random variable as linear combinations of the orthogonal polynomials:

$$
f(z) \approx \sum_{i=1}^{N} c_i \varphi_i(z) \tag{4}
$$

At this point, collocation methods are used to form a system of linear equations to solve for the coefficients, c_i. Clearly, the amount of solution points needed is equal to the number of polynomials used to form this surrogate, N. The collocation points can be selected by several different techniques, but for this study they were chosen to be the roots of the higher order orthogonal polynomials. This selection is optimal for the evaluation of the integral form of the expectation operator. For a function of a random vector, the polynomials are defined in [7]. The function can be expressed as a combination of these polynomials in a similar manner to (4).

3.4. *Error Estimates*

To evaluate the validity of the surrogate model, the forward model must be sampled again and compared to the surrogate model by some means. For this study, these points were

chosen as the 2nd higher order polynomial roots for the sake of computational savings in the event that the approximation is not adequate at the current polynomial order. The solutions of the forward model and the polynomial surrogate can be compared with the probabilistic sum of the square residuals:

$$ssr = \sqrt{\frac{\sum_{i=1}^{n} \epsilon_i}{n\rho(r_\mu)}} \tag{5}$$

$$\epsilon_i = \left(u_{app,i} - u_{m,i}\right)^2 \rho(r_i)$$

or the relative measure:

$$rssr = \frac{ssr}{E[u_{app}(r)]} \tag{6}$$

In these measures, $\rho(r)$ is the joint probability distribution function of the random vector, r_μ is a vector of the means of the random parameters, $u_m(r)$ is the forward model and $u_{app}(r)$ is the surrogate model.

4. Case Study

The specific problem for this study is that of detecting flaws in the edges of samples with ECT methods. The problem presents several challenges experimentally making model validation with benchmark data relatively difficult. There are semi-analytical solutions for this type of problem [9] but due to availability, a basic finite element model was created for this study, and a mesh convergence study is considered to be adequate validation. The program used to model the problem is COMSOL Multiphysics. Similar problems solved with COMSOL have been previously experimentally validated [10]. Since the field equations and boundary conditions have not changed, it is assumed that the mesh convergence study is adequate for validation.

4.1. Problem Setup: Field Equations

The field equations for the eddy current problem come from a time-harmonic magnetic vector potential solution to Maxwell's equations:

$$\frac{1}{\mu}\nabla \times \nabla \times \boldsymbol{A} + j\omega\sigma\boldsymbol{A} + \sigma\nabla v = \boldsymbol{J^e} \tag{7}$$

The solution is iterated for multiple probe positions, and the solution for the fields are used in equation (8) to calculate the change of resistance and inductance.

$$\mathrm{R} = \frac{1}{\mathrm{I}^2}\int_{\Omega'} \mathbf{JE}^*\,\mathrm{d}\Omega', \qquad \mathrm{L} = \frac{1}{\mathrm{I}^2}\int_{\Omega} \mathbf{HB}^*\,\mathrm{d}\Omega \tag{8}$$

Figure 1. Solutions for change in resistance and reactance as the coil is run over the edge notch.

An example of the results from the virtual scan is shown in Figure 1. Results from a mesh convergence study are shown in Figure 2. Clearly the solution, |Z|, is converging with respect to increasing mesh size.

4.2. The Forward Model in the Stochastic Framework

For the purposes of this study, the conductivity and notch depth were considered random parameters. The conductivity was chosen to include a variable that should have controlled outcomes to prevent negative values. Varying notch depth implies that the geometry of the problem is changing, which results in mesh errors in the solution vector in the linear system. Since these errors directly result in errors in the surrogate model, the error measures had to be relaxed, but the approximation is still shown to be adequate for the purposes of the study. The mean of the distributions of resistance and reactance changes are shown calculated with several different methods. The mean calculated from the Monte Carlo simulation of the surrogate model is very close to the other methods, showing that the surrogate model is at least valid for moment estimation. The PDF's of these solutions are shown in Figure 3, but no validation of these curves has been performed as of yet.

5. Conclusions, Discussions, and Future Work

The PCM was applied to a standard eddy current problem which was modeled with FEM. The surrogate model for the FEM solution converged at 3rd order when the convergence constraints were relaxed. The most effective method of avoiding the need to relax the convergence constraints would be to disregard geometry uncertainties, but since the edge

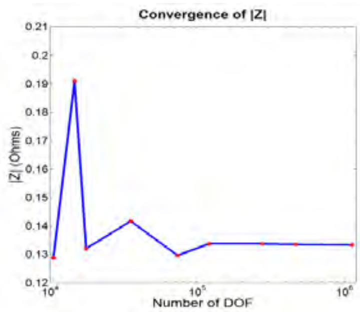

Figure 2. Results from the convergence study clearly show convergence of the finite element model

Figure 3. The PDF's of both resistance and reactance from the surrogate model

notch problem is very sensitive to geometry conditions, this is not feasible. If the mesh were made finer, the errors in the solution would reduce, but the computation time could potentially grow out of hand. Another method could be to change the method of selecting collocation points, but this analysis is left for future work. Even with a relaxed convergence criteria, the surrogate model still seems to estimate the moments of the distributions adequately, which serves as an initial validation of the method for eddy current problems. A numerical way of estimating the validity of PDF's produced with this method is still needed, but this again is left for future studies.

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